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OPTIMIZATION OF THE PROCESSES OF TREATMENT OF MATERIALS WITH SUPPLEMENTATION OF THE DEFINITION OF UNKNOWN RANDOM INITIAL CHARACTERISTICS

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Consideration is given to the issues of optimization of the processes of treatment of materials with certain unknown initial characteristics. Supplementing of the definition of these unknown parameters can be considered as adaptation of a mathematical model.

From a mathematical standpoint, a search for a rational process of treatment of materials is a problem of stochastic optimization (PSO).

Depending on the specific conditions and on the objectives of investigation, the problem is most frequently reduced to one of the following variants: (a) a search for a rational solution on the average (the M model); (b) minimization of the spread in the solution (the D model); (c) minimization of the probability of a deviation of the solution from a certain prescribed value (the P model); (d) finding the optimum solution for the worst distribution of initial data (the MM model). The letters used in the models' names denote: M, mathematical expectation, D, dispersion, P, probability, MM, minimum of the maximum or maximum of the minimum. A procedure of solving the problem for various types of models is described in [1] in detail.

Under the conditions of actual production, interest is frequently provoked by a certain combination of the models rather than by specific models [1]. The problem is reduced to a multicriterial problem of stochastic optimization, for which we must construct a complex figure of merit. An approach to the construction of this criterion is proposed that leans upon the prerequisite that fuzzy sets rather than numbers are the elements of investigation [2]. We will call it a complex optimization criterion (COC).

By a fuzzy set A is meant a combination of pairs of the form $(u, \mu_A(u))$, where $u \in U$, U is the set of elements in the ordinary sense ($U \subset R$), and $\mu_A(u)$ is the membership function of the fuzzy set A that is defined as $\mu_A : U \to [0, 1]$.

The expression $\mu_A(u) = 1$ means full membership of the element u in the set U, while $\mu_A(u) = 0$ indicates that the element u does not belong to the set U. For an arbitrary element u, the membership function governs the degree of its belonging to the set U. If the corresponding models (M, D, P, and MM) are selected as the elements of a fuzzy set, then, by the above definition, the fuzzy set A consists of four pairs and can be written in the form

$$A = \bigcup_{i=1}^{4} \mu_A(a_i)/a_i . \tag{1}$$

We note that if the fuzzy set A is represented as a random quantity defined on the set R we can select the distribution density or the distribution function A as the membership function.

Now the PSO can be formulated as follows.

We must determine the control vector that would ensure the minimum value of the objective function on the fuzzy set A for all the equality and inequality constraints. Equality constraints describe a boundary-value problem of treatment of materials, while inequality constraints extend to all the state and control variables.

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To solve the PSO in the formulation presented, we must determine a procedure for comparing fuzzy numbers on the fuzzy set. The so-called ranking indices that permit comparison of fuzzy numbers are introduced in [3].

For further consideration, we must introduce the concept of the carrier of a fuzzy set (the carrier of a fuzzy number is similarly defined).

The carrier of the fuzzy set A refers to the set $S(A)$ that is defined as follows:

$$S(A) = \{a \,\|\, a \in U, \mu_A(a) > 0\}. \tag{2}$$

In the general case, the order relation (of a "larger," "smaller," or "equal" type) for fuzzy numbers is fuzzy itself. Only where the intersection of the carriers of the fuzzy numbers A_1 and A_2 is empty will the order relation be precise. Therefore, we must define a certain precise function of fuzzy arguments that, regardless of the relation of the carriers of fuzzy numbers, would unambigously determine the order relation between them. In the work, we propose several procedures for calculating this precise function $H(A, B)$ of the fuzzy arguments A and B that is referred to as the ranking index. The value of the latter for a specific pair of fuzzy arguments leads us to solving the question as to which of the two numbers is smaller. For example:

$$H_1(A, B) = \sup_{a \geq b} \min\{\mu_A(a), \mu_B(b)\}, \tag{3}$$

in this case, if $H_1(A, B) \geq H_1(B, A)$, then $A \geq B$.

The index H_1 separates out the fuzzy number A_i as the largest one in which $\sup \arg \sup\limits_{a \in S(A_i)} \mu_A(a)$ is the largest (i.e., the number in which the maximum of the membership function is located more to the right along the U axis).

The ranking indices of the type (3) are used for fuzzy sets characterized by fuzzy numbers with elements of the same scale. Therefore, employing them to construct a fuzzy set that characterizes a complex figure of merit (this fuzzy set will be denoted by the superscript r) required redefinition of the corresponding elements.

In particular, if the mathematical expectation of the solution is selected as a_1, we must select the root-mean-square deviation as a_2, the mathematical expection of the solution in the case where the probability of deviation of the solution from a certain prescribed region would be minimum as a_3, and the worst value of the solution for the most unfavorable distribution of the initial parameters as a_4.

To avoid this redefinition, we construct a fuzzy set whose elements will be relative and dimensionless magnitudes of the criteria rather than their absolute values. By virtue of the constant significance of each corresponding component of two fuzzy numbers A^r and B^r (with all possible controls), the ranking index can be constructed as follows:

$$H(A^r, B^r) = \operatorname{sign} C_i, \tag{4}$$

where $C_i = \mu_i(a_i^r - b_i^r)/d_i$, i conveys $\max\limits_i |\mu_i(a_i^r - b_i^r)/d_i|$, μ_i is the membership function (significance) of a_i^r (or b_i^r), $d_i = \max(a_i^r, b_i^r)$, $i \in \overline{[1, n]}$, and n is the number of pairs that determine the fuzzy set. If the value of i is unique and if sign C_i = "+", then $A^r > B^r$; if sign C_i = "−", then $A^r < B^r$.

If the value of i is not unique, we determine k, i.e., the number of extrema equal in modulus ($k \leq n$), and calculate

$$\lambda = \sum_{i=1}^{k} \operatorname{sign} C_i. \tag{5}$$

If $\lambda = 0$, then $A^r = B^r$; if $\lambda < 0$, then $A^r < B^r$; if $\lambda > 0$, then $A^r > B^r$.

We note that μ_i is essentially expert evaluations. By virtue of their fuzziness we can solve the multicriterial problem of stochastic optimization with the use of the complex figure of merit and the ranking index of the type (3) and (4) at $\mu_i = 1$, $i = \overline{1, n}$. Furthermore, the above procedure for solving the problem can be used to advantage for a problem of multicriterial optimization and in a determinated case.

Ranking indices of the type (3), (4), and (5) are determinate in a sense, since each separates out for comparison a certain value of the fuzzy number with the corresponding membership function, and their use is justified when this value can be separated out by any sign.

There are ranking indices of integral character. It is convenient to use them for fuzzy numbers with approximately the same carriers and with membership functions that are rather difficult for mutual comparison. In this work, we used a ranking index of the form

$$H_5(A, B) = H_+(A) - H_+(B)\,, \quad H_+(A) = \int_0^1 M(A_\alpha)\, d\alpha\,, \tag{6}$$

where A_α is the α-level set of the fuzzy number A, i.e.,

$$A_\alpha = \{a\colon \mu_A(a) \geq \alpha\}\,; \quad M(A_\alpha) = (a^0 + a^+)/2\,,$$

where $a^0 = \inf\limits_{a \in A_\alpha} a$ and $a^+ = \sup\limits_{a \in A_\alpha} a$. If $H_5(A, B) \geq 0$, then $A \geq B$.

The integral ranking indices take into account the entire range of definition of fuzzy numbers.

In many cases, to solve the PSO, we must supplement the definition of unknown characteristics of the parameters that are described by random quantities. The process of supplementing the definition can be considered as adaptation of the mathematical model.

The modeling and optimization scheme for strain processes under the conditions of indefiniteness of parameters and uncertainty of some of their characteristics can be written in terms of optimum adaptive systems. In the class of problems under investigation, the elements of the assumed scheme of the adaptive system are:

(a) the object of control – a mathematical model of a certain technological process that must be optimized in accordance with a prescribed figure of merit;

(b) the control device – the control and state variables of the PSO;

(c) the adapter – the algorithm of supplementing the definition (assessment) of lacking information on the control and state variables;

(d) the control device in a feedback path – the algorithm of the optimization method employed.

In accordance with the adopted terms, by a basic loop we will mean one method of solving the PSO and by an adaptation loop – the procedure for supplementing the definition of unknown characteristics of the parameters of the process.

Within the framework of the description of undefined parameters by random quantities that is adopted in the work, a general problem of supplementing the definition (PSD) of unknown parameter distributions can be formulated as follows.

Let a functional dependence of the random quantity Y on the system of random quantities $\mathbf{X} = (X_1, X_2, ..., X_n)$ be known: $Y = \varphi(X_1, X_2, ..., X_n)$. Let I and $\bar{I}$ be two nonintersecting sets of indices; $I \cup \bar{I} = \{1, 2, ..., n\}$. Let the distributions of the random quantities Y and X_i ($i \in I$) also be known. It is necessary to find the distribution density of the random quantities X_k ($k \in \bar{I}$).

In this work, consideration is given to the processes for which PSD can be reduced to a partial case (PCPSD). We formulate the PCPSD.

Let a functional dependence of the random quantity Y on the system of random quantities $\mathbf{X} = (X_1, X_2, ..., X_n)$ be known: $Y = \varphi(X_1, X_2, ..., X_n)$. The distributions of the random quantities Y and X_k, $k = \{1, 2, ...,$

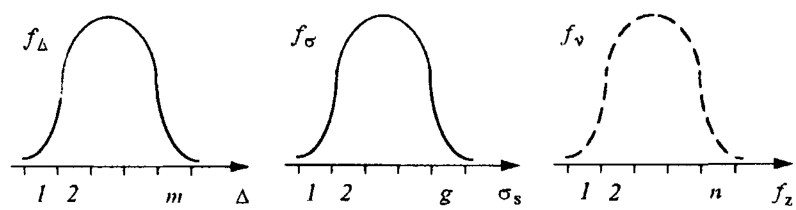

Fig. 1. Representation diagram of the probability densities; the dashed curve is the unknown distribution; the figures on the abscissa axis are the numbers of intervals.

$i-1$, $i+1$, ..., $n\}$, are also known. Considering the random quantities X_k to be independent, we must find the distribution density of the random quantity X_i.

Below, we propose two methods for evaluating numerically an unknown distribution of the random quantity X_i: a computational method and a method of hypotheses.

The main idea of a computational method consists of obtaining the distribution of X_i using some relations of probability theory and data on the distribution of Y and of the remaining components of **X**. It is applicable in the absence of information on the distribution of X_i.

The main idea of the method of hypotheses lies in the fact that the hypothesis of the form of the unknown distribution of X_i is adopted a priori, and then the optimization problem that minimizes a deviation of the obtained distribution of Y from a prescribed one is solved. In this case, the components of the control vector are the characteristics of the distribution of X_i. This method is employed in the case where the form of the X_i distribution is known but its parameters are unknown.

We consider these methods in greater detail, using the problem of upsetting of a cylindrical specimen as an example. The formulation, procedure, and some results of the solution of this problem are presented in [4].

Let the plastic strain resistance σ_s and the Sybel friction coefficient f_z be stochastic in this problem. Then the deviation of the lateral surface of the produced workpiece from a prescribed Δ is a function of σ_s and f_z:

$$\Delta = \varphi(\sigma_s, f_z),$$

where σ_s and f_z have a stochastic spread. Before solving the PSO we must determine the families of distributions for σ_s and f_z and evaluate their specific parameters. The distribution of σ_s can be obtained based on laboratory statistical investigations. It is much more difficult to evaluate the distribution of f_z by virtue of the sophistication of the experiments. Therefore, the problem of searching for methods of numerical evaluation of the f_z distribution is urgent.

For the example under consideration, the computational method is as follows. It is assumed that the distribution densities f_Δ and f_σ for, respectively, Δ and σ_s are known, and the interval of spread in f_z is also known. The distribution density f_ν for the friction coefficient f_z is unknown; it must be found (from here on, we will use the subscript σ for the quantities that refer to σ_s and the subscript ν for the quantities related to f_z). For this purpose, the corresponding regions of definition of the functions that describe the distribution densities are divided into intervals (Fig. 1), where n and g are arbitrary integers and $m = n-1$.

Each interval corresponds to the average values

$$c_i^\Delta = \frac{\Delta_i + \Delta_{i+1}}{2}, \quad c_k^\sigma = \frac{\sigma_{s,k} + \sigma_{s,k+1}}{2}, \quad c_j^\nu = \frac{f_{z,j} + f_{z,j+1}}{2}$$

and probabilities

$$p_i^\Delta = \int_{\Delta_i}^{\Delta_{i+1}} f_\Delta d\Delta, \quad p_k^\sigma = \int_{\sigma_{s,k}}^{\sigma_{s,k+1}} f_\sigma d\sigma_s, \quad p_j^\nu = \int_{f_{z,j}}^{f_{z,j+1}} f_\nu df_z.$$

Considering σ_s and f_z to be independent random quantities and employing the elementary relations of probability theory, we can obtain the system of linear equations

$$p_1^{\Delta} = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n ,$$

$$p_2^{\Delta} = a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n ,$$

$$\ldots$$

$$p_{n-1}^{\Delta} = a_{(n-1)1}x_1 + a_{(n-1)2}x_2 + \ldots + a_{(n-1)n}x_n , \tag{7}$$

$$1 = x_1 + x_2 + \ldots + x_n ,$$

where x_j is the sought probabilities p_j^{v}; $a_{ij} = \sum_k p_{kj}^{\sigma}$; the elements of the sum are only p_{kj}^{σ} that correspond to such c_k^{σ} that satisfy the condition $\varphi(c_k^{\sigma}, c_j^{\mathrm{v}}) \subset [\Delta_i, \Delta_{i+1}]$.

However, this method turned out to be of little use, since the last, normalizing equation imposes no constraints on the positiveness of x_j. As a result, the solution of system (7) can involve negative numbers. To avoid this, we propose another variant of the computational method – solution of the canonical problem of linear programming

$$\min\,(\mathbf{c}, \mathbf{x}) , \quad \mathbf{Ax} = \mathbf{b} , \quad x_j \ge 0 , \quad j = \overline{1, n} ,$$

where $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{A}$ is an $m \times n$ matrix, $\mathbf{b} \in \mathbf{R}^m$, and $\mathbf{c} \in \mathbf{R}^n$. In the example under consideration, $\mathbf{A}$ is the matrix with elements a_{ij}, $\mathbf{c}$ is the unit vector, $\mathbf{b}$ is the vector with the coordinates p_i^{Δ}, $i = \overline{1, m}$, and $\mathbf{x}$ is the vector whose coordinates are the sought probabilities p_j^{v}, $j = \overline{1, n}$. To solve this problem, use is made of a modified simplex method.

The method of hypotheses is based on the a priori hypothesis of a family of sought distributions. In the example under consideration, the form of the f_z distribution is hypothesized based on primary data of physical character. Next, the optimization problem is solved, in which the control variables are the parameters of the f_z distribution and the objective function is the deviation δ of the obtained distribution from a prescribed distribution of the quantity Δ: $\delta = \|f_{\Delta} - f_{\Delta}^{*}\|$. As the objective function for "measuring" the deviation we can use the chi-square (χ^2) criterion, the Chebyshev norm, etc. When the obtained and prescribed distributions of Δ are rather close, the hypothesis of the form of the f_z distribution is adopted and the parameters obtained by solution of the optimization problem are used as the distribution ones.

It should be noted that the term "closeness of distributions" from the viewpoint of a norm in the general case does not yet mean the correspondence of the form of the obtained distribution to the prescribed form of the distribution of Δ. Therefore, it seems necessary to check the form of the obtained distribution of Δ. For this purpose, we can employ any permissible criterion, for example, χ^2.

The development of this method is a simultaneous check of several hypotheses of the family of f_z distributions followed by the selection of the best of them in terms of the minimum δ.

We give the results of employing the computational method to solve the PCPSD in the case of drawing a tube [5] by the dependence $\Delta = \varphi(R_1, \sigma_s)$ where Δ is the deviation of the obtained thickness of the tube from a prescribed one; R_1 is the initial (external) radius of the billet.

We are given:

(1) $R_1 \sim N$ (18.870 mm; 0.005 mm) and $\Delta \sim N$ (0.222 mm; 0.02 mm).

(2) $\sigma_s \in [62; 68]$ MPa.

We must find the distribution of the plastic strain resistance σ_s.

Let $m = 5$ and $n = 14$.

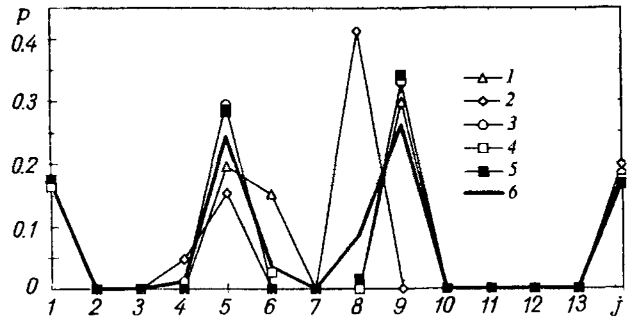

Fig. 2. Sought values of p_j^{σ} for different g: 1) 20; 2) 30; 3) 50; 4) 100; 5) 150; 6) average one.

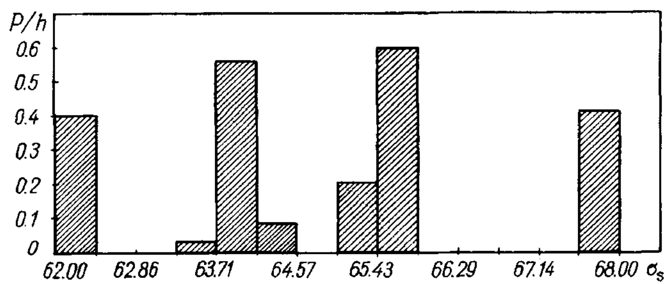

Fig. 3. Histogram of the distribution of σ_s, MPa.

The solution results for different $g = \{20, 30, 50, 100, 150\}$ (g is the number of intervals into which we divide the region of definition of the function that describes the distribution density of the parameter R_1) are presented in Fig. 2. From the average values of p_j^{σ} and the corresponding average values of the segments c_j^{σ} of the interval of spread in σ_s, a histogram of the sought distribution is constructed (Fig. 3).

Analysis of the given plots shows the following:

(1) in employing the simplex method, some p_j^{σ} are inevitably zero; therefore, the obtained distribution can have a complex form;

(2) the solution found in the form of a combination of pairs (c_j^{σ}; p_j^{σ}), $j = \overline{1, n}$, can directly be used in problems where the distribution densities of the parameters are approximated by histograms.

We give the results of employing the method of hypotheses to solve the PCPSD in the case of precision stamping [6].

We take

$$\delta = \|f_\eta - f_\eta^*\|, \quad \eta = \frac{W}{k_W} + \frac{|2x_2 - s^*|}{k_S},$$

where k_W and k_S are the normalizing coefficients, s^* is the prescribed size, W is the work done by the tool on the movement ΔL, and $W = \int_{\Delta L} Q(z)dz$.

Let η be a function of the initial height of a cylindrical billet L_0 and the Coulomb friction factor f:

$$\eta = \varphi(L_0, f),$$

where L_0 and f have a stochastic spread; the distribution of f is unknown and it must be evaluated.

We are given:

(1) $L_0 \sim N$ (19.05 mm; 0.01 mm) and $\eta \sim N$ (0.967; 0.019);

TABLE 1. Results of Solving the PCPSD by the Method of Hypotheses with the Use of the Criterion χ^2 and the Chebyshev Norm

Assumed distribution of f	q	Number of steps	mf	σf	$m\eta$	$\sigma\eta$	χ^2	δ
				Criterion χ^2				
Normal	5	85	0.267	0.020	0.968	0.011	2.169	12.634
Uniform	5	85	0.267	0.020	0.968	0.014	3.144	3.325
Normal	7	89	0.267	0.020	0.967	0.011	1.652	18.738
Uniform	7	84	0.267	0.020	0.967	0.013	8.165	6.74
Normal	10	92	0.267	0.020	0.967	0.011	2.339	31.891
Uniform	10	87	0.267	0.020	0.967	0.012	20.856	13.999
				Chebyshev norm				
Normal	5	15	0.224	0.011	0.944	0.006	1.890	20.988
Uniform	5	30	0.100	0.005	0.874	0.003	5.106	20.954
Normal	7	15	0.230	0.011	0.946	0.006	1.364	20.989
Uniform	7	13	0.190	0.011	0.924	0.007	9.500	20.980
Normal	10	15	0.230	0.011	0.945	0.006	4.723	20.989
Uniform	10	14	0.240	0.011	0.951	0.006	24.758	20.988
Normal	15	15	0.230	0.011	0.944	0.006	27.656	20.989
Uniform	15	15	0.240	0.011	0.951	0.007	48.234	20.989
Normal	20	15	0.230	0.011	0.944	0.007	73.193	20.989
Normal	20	15	0.240	0.011	0.951	0.007	89.058	20.989

(2) the mathematical expectation $mf \in [0.1; 0.3]$ and the root-mean-square deviation $\sigma f \in [0.005; 0.020]$;

(3) hypotheses of normal and uniform distributions of the friction factor f.

Let us find the best one of the hypotheses in terms of the minimum δ.

Solutions obtained with the employment of the criterion χ^2 and the Chebyshev norm are presented in Table 1, where q is the number of segments into which the regions of definition of the distributions of L_0, f, and η were divided.

From the data presented in the table we can draw the following conclusions.

(1) When the criterion χ^2 is employed, the deviation δ, regardless of q, is smaller for the case where the hypothesis of a uniform distribution of f was considered. When the Chebyshev norm is employed, δ is practically the same for normal and uniform distributions of L_0. Therefore, the employment of the Chebyshev norm in the example under consideration provides no answer to the question of which of the hypotheses is more preferable. The hypothesis of a uniform distribution of σ_s can be adopted based on the data from Table 1 obtained with the employment of the criterion χ^2.

(2) In all the cases, the value of the criterion χ^2 confirms the hypothesis that the obtained distribution of η can be considered to be normal with the corresponding parameters indicated in the table.

Of special interest is the solution of the PCPSD, in which the distributions of several initial parameters are unknown. We consider this case using the problem of upsetting as an example; the distributions of the plastic strain resistance σ_s and the Sybel friction factor f_z are considered to be unknown. We will use the method of hypotheses, considering that both normal and uniform distributions are possible. The results of solving the problem are presented in Table 2.

We revert to the solution of the PSO for the process of upsetting.

TABLE 2. Solution of the PCPSD with Unknown Distributions of Several Initial Parameters

Assumed distributions of f_z and σ_s	Number of steps	mf_z	σf_z	$m\sigma_s$, MPa	$\sigma(\sigma_s)$, MPa	$m\Delta$, mm	$\sigma\Delta$, mm	χ^2	δ
$f_z \sim N$ and $\sigma_s \sim N$	714	0.298	0.030	47.266	1.500	4.912	0.091	2.894	1.703
$f_z \sim R$ and $\sigma_s \sim N$	459	0.313	0.030	46.309	1.500	4.911	0.095	0.975	1.100
$f_z \sim N$ and $\sigma_s \sim R$	790	0.250	0.022	50.761	1.492	4.930	0.088	2.119	1.366
$f_z \sim R$ and $\sigma_s \sim R$	761	0.252	0.020	50.549	1.500	4.916	0.098	0.535	0.524

The solution of the PCPSD yielded that f_z is distributed by the normal law. Next, for calculations, use was made of the following initial data: $\sigma_s \sim N$ ($m\sigma_s$, 15 MPa), $m\sigma_s \in$ [400, 560] MPa; $f_z \sim N$ (mf_z, 0.03), and $mf_z \in$ [0.09; 0.4]. Here $m\sigma_s$ and mf_z are the mathematical expectations of, respectively, the plastic strain resistance σ_s and the friction factor f_z. To solve the optimization problem, we employed the Nelder–Mid method.

The characteristics (objective functions) for the problem of upsetting were: a_1, mathematical expectation of Δ; q_2, root-mean-square deviation of Δ; a_3, probability of the fact that Δ will not exceed a certain prescribed value (4.6 mm); a_4, maximum value of Δ along the entire length of the generatrix of the lateral surface.

Solving the problem of selection of the optimum upsetting regime with different approaches to the construction of the objective function permitted the following results.

1. The objective function is an individual model:

A model (determinate analog with $\sigma_s = m\sigma_s$ and $f_z = mf_z$): $m\sigma_s$ = 400.0 MPa; mf_z = 0.159;

M model: $m\sigma_s$ = 400.0 MPa; mf_z = 0.183;

D model: $m\sigma_s$ = 486.6 MPa; mf_z = 0.243;

P model: $m\sigma_s$ = 400.0 MPa; mf_z = 0.212;

MM model: $m\sigma_s$ = 412.8 MPa; mf_z = 0.249.

2. The objective function is a linear combination of objective functions for individual models with significance coefficients $\mu_1 = 0.8$, $\mu_2 = 0.5$, $\mu_3 = 0.3$, and $\mu_4 = 0.7$ for a_1, a_2, a_3, and a_4, respectively. We obtained the following regime: $m\sigma_s$ = 411.7 MPa; mf_z = 0.213.

3. The objective function is the fuzzy set A with degrees of membership $\mu_1 = 0.8$, $\mu_2 = 0.5$, $\mu_3 = 0.3$, and $\mu_4 = 0.7$ for a_1, a_2, a_3, and a_4, respectively. For comparison, the index $H_1(A, B)$ was used. We obtained the following regime: $m\sigma_s$ = 400.0 MPa; mf_z = 0.183.

4. The objective function is the fuzzy set A^r with degrees of membership $\mu_1 = 0.8$, $\mu_2 = 0.5$, $\mu_3 = 0.3$, and $\mu_4 = 0.7$ for a_1^r, a_2^r, a_3^r, and a_4^r, where a_1^r coincides with a_1; a_4^r coincides with a_4; a_2^r is the variance of the deviation; a_3^r is the probability of the deviation exceeding a value of 4.6 mm. For comparison, the index $H(A^r, B^r)$ is used. We obtained the following regime: $m\sigma_s$ = 410.3 MPa; mf_z = 0.227.

5. The objective function is similar to the objective function defined in item 3. For comparison, the index $H_5(A, B)$ is used. For the membership functions $\mu_1 = 0.8$, $\mu_2 = 0.5$, $\mu_3 = 0.3$, and $\mu_4 = 0.7$, we obtained the following regime $m\sigma_s$ = 404.6 MPa; mf_z = 0.186.

6. The objective function is the random quantity A, whose distribution histogram is determined by the histograms of stochastic initial parameters (σ_s and f_z). For comparison, the index $H_1(A, B)$ was used. We obtained the following regime: $m\sigma_s$ = 400.0 MPa; mf_z = 0.159.

7. The objective function is similar to the objective function defined in item 6. For comparison, the index $H_5(A, B)$ is used. We obtained the following regime: $m\sigma_s$ = 400.0 MPa; mf_z = 0.159.

Comparing the solutions given in items 6 and 7, we can draw the conclusion of practical coincidence of the results. This is due to the fact that in the case under investigation for a normal distribution of σ_s and f_z the distribution density function is a function with a pronounced maximum and a rather large excess. The result obtained here with the employment of the index $H_5(A, B)$ is natural.

8. The objective function is the criterion χ^2 that verifies the hypothesis of a normal distribution of the solution. In this case, we do not merely compare the random quantities A_i and A_{i+1} obtained in optimization but first determine the values of the criterion χ^2 for A_i and A_{i+1} and only after that compare χ_i^2 and χ_{i+1}^2. We obtained the following regime: $m\sigma_s$ = 439.1 MPa, mf_z = 0.284, and χ^2 = 0.154.

The obtained value of χ^2 suggests that the found control ensures the acceptability of the proposed hypothesis of a normal solution distribution with the level of significance α = 0.9.

The above results enable us to state:

(a) The solution of the PSO depends significantly on the form of the objective function.

(b) All other things being equal, the selection of the ranking index determines, in many respects, the selection obtained for the PSO.

(c) All other things being equal and when the same ranking indices are employed, the solution depends on the structure of the fuzzy set selected as the objective function.

(d) The use of one optimality criterion or another in solving the PSO depends on the actual conditions and objectives of the investigation.

Thus, for example, from a technologist's viewpoint, the M model can be employed in the case where the objective size has a significant tolerance and it will suffice to ensure the maximum possible approximation of this size to the middle of the tolerance zone; the D model is suitable when the tolerance zone of the objective size is small.

In mass and large-scale productions, the possibility exists of obtaining large statistical samples. Under these conditions, it can turn out to be expedient to use the P model since, by definition, it minimizes the probability of a deviation of the solution from a certain prescribed value that can be evaluated with a high degree of accuracy given abundant experimental data. And, conversely, in individual and small-scale production, it is expedient to employ the MM model to obtain the best one of the worst solutions because of the absence of a sufficient body of experimental data.

(f) In comparing fuzzy numbers, we can use both "determinate" and integral ranking indices. It should be borne in mind that integral indices enable us to allow for the entire range of variation in the fuzzy number, while "determinate" ones separate out the elements with the extremum, in a sense, value of the membership function.

CONCLUSIONS

1. Mathematical models for constructing generalized criteria of optimization (objective function) for the problem of stochastic optimization with one or several criteria are developed. The theory of fuzzy sets and probability theory were used for the investigation.

2. Two procedures for determining the distribution density of one initial parameter of the problem with the known distributions of the remaining initial parameters and of the solution are proposed. The problem under investigation is a partial case of the problem of adaptive control.

3. The results of solving the PCPSD for the processes of upsetting of a cylindrical specimen, drawing of a tube, and precision stamping are given.

4. Rational regimes for upsetting a cylindrical specimen with different objectives of investigation are recommended.

NOTATION

A and B, random quantities, fuzzy sets, and matrices; $S(A)$, carrier of the fuzzy set A; $H(A, B)$, ranking index; μ_A, membership function of the fuzzy set A; a_i and μ_i, element and the corresponding value of the membership function of the fuzzy set A that is a complex figure of merit; σ_s, plastic strain resistance; f_z, Sybel friction factor; f, Coulomb friction factor; p, probability; h, segment length along the abscissa axis; χ^2, chi-square criterion; Δ, deviation of the lateral surface of the produced workpiece from the prescribed one; δ, deviation of the obtained distribution from the prescribed distribution of the random quantity; η, figure of merit for the problem of precision stamping; f_Δ, f_σ, f_V, and f_η, distribution densities of, respectively, Δ, σ_s, f_z, and η; $m\sigma_s$ and mf_z, mathematical expectations of, respectively, σ_s and f_z; $N(m;\ \sigma)$, normal distribution with the mathematical expectation m and the root-mean-square deviation σ; $f_z \sim N$, quantity f_z is distributed by the normal law; $f_z \sim R$, quantity f_z is distributed by the uniform law. Subscripts and superscripts: r, randomness; s, sliding.

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